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FOCUSING ON PROVING SKILLS FOR SENIOR SECONDARY TWO STUDENTS IN CIRCLE GEOMETRY

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Abstract

The learning of circle theorems is frequently highlighted as one challenging geometric content area in the Nigerian Mathematics curriculum for Senior Secondary two students. However, it remains unclear how students, are reasoning when dealing with proving of circle theorems. This paper rigorously demonstrates the fundamental relationships between angles, lines, and arcs within a circle, often using geometric proofs. It provides a comprehensive and accessible proving skills which are fundamental for circle theorems. The skills utilize geometric principles and constructions, aiming for clarity and ease of understanding. It presents a comprehensive and accessible set of proofs for key circle theorems, through detailed geometric constructions and logical reasoning, making the concepts understandable for students' achievement examinations. It covers key concepts such as angles at the centre and circumference, angles in a semicircle, and cyclic quadrilaterals, along with the geometric reasoning or statement step-by-step behind them.

Key words: Proving Skills, Circle Geometry, Proof, Mathematics and Geometry

Introduction

Mathematics is considered a vital subject in the school curriculum worldwide, which has strong connections with other subjects such as science and technology. It affects all aspects of human life, including economics, politics, geography, science and technology. It is therefore, considered as the backbone for any meaningful scientific endeavour. For any nation to advance in science and technology, conscious efforts must be deployed towards empowering the youths with the necessary Mathematics skills. It consists of numerous domains that continue to develop in a cumulative manner towards the society. Therefore, students need to acquire mathematical competence at all levels of education. It is common knowledge that the major branches of Mathematics are Arithmetic, Algebra, Geometry, Trigonometry, Statistics, Calculus, and Discrete Mathematics.

Geometry is a branch of Mathematics and important part of the Mathematics' curriculum with a wide range of applications in daily life. Its rich history dates back to ancient Egypt, where it was used to measure land, and the Greeks studied the properties of shapes (Cooke, 2007). As well, geometry is crucial to the study of other branches of mathematics. According to Choudhury and Das (2012), geometry is a combining theme to the entire Mathematics curriculum and as such is a rich foundation of visualization for arithmetical, algebraic and statistical concepts. In the Nigerian senior secondary school Mathematics curriculum, geometry spread all over the educational levels based on its relevance and application in the other areas of Mathematics and the society. It constitutes 38% of the mathematics curriculum and it is the worst performed topic in all external and internal examinations making the overall subject achievement very poor (Nwafor, Tsoho, & Usman, 2023).

A circle is an example of plane geometry. The circle is a shape that is been used frequently in daily lives: wheels, dinner plates, cups and bottle caps are all circles. Learning about circles is a necessary element of understanding geometry. A circle is a simple figure encircled by a curved line, each point of which is equidistant from a central point (Hussin, et al. 2023). Circle geometry is also called circle theorem is a sub-topic in geometry subject or a theoretical aspect of geometry concept that requires proof of the theorems. This aspect of geometry (circle) has been difficult for teachers and learners both to teach and to learn. The factors contributing to these are abstract geometry, learning methods still dominated using algorithms and formulas, students' inadequate spatial abilities, and geometric language (Monique.& Nfor, 2023). As a result, learners are unable to construct, visualise, and justify the

circle theorems. Therefore, most learners, even teachers memorised the theoretical aspect of geometry which has been a problem in teaching and learning the concept (Tay & Wonkyi, 2020). Research studies showed that this theoretical aspect of geometry (circle theorems) is being taught using proving skills (Ákos, & Kónya, 2022).

Despite the importance of Mathematics, students in developing countries constantly do not perform well in the subject (Akugizibwe & Ahn, 2020; Ekok, Owan & Ubugha,2024). In Nigeria, Mathematics has become a subject that students dread the most. According to the Chief Examiners' Annual Reports of the West African Examination Council (WAEC), the students who had credit passes in Mathematics was 33.19% in 2024. They also maintained that students have weaknesses in different areas of geometry, such as mensuration, constructions, and proving circle theorem that required proving skills. They also lack the proving skills in circle geometry. Again, the students' performance in SSSCE Mathematics (Core), indicated most candidates avoided the questions on circle theorems and the few who attempted it; many were unable to solve the problem accurately because they were not able to write correct logical skills needed for circle theorems. Apparently, many students preparing for their West African Senior School Certificate Examination in Mathematics cannot boast of being confident in the fundamental concepts of circle theorems. This may be attributed to students' inadequate knowledge of circle theorems. It is the task of the teachers of Mathematics to help their students to understand and develop confidence in the proving skills of these circle theorems (Aidoo & Akayuure, 2022). It is often associated with the study of abstract concepts such as non-dimensional points and lines. Only imagination can create these figure(s). It is a visual subject; thus, it is hard to conceive thinking geometrically without drawing lines or utilizing a range of lines to describe an abstract geometric concept. Learners frequently struggle to understand how different geometric shapes are represented (Folake, Yeukai, & Jogymol,2024). Studies and experiences showed that the most difficult part of geometry education is the development of students' reasoning, proving skills and about 95% of the students did not answer the questions on circle geometry (Ákos, & Kónya, 2022; Bruna, 2023).

However, when students' performance in Mathematics was further analysed into specific aspects of Mathematics to determine which areas students had problems to aid better understanding, it was discovered specifically that students performed poorly in proving circle theorems in geometry (Akanmu & Bala, 2022; Esuong et al., 2024; Wakhata et al., 2022) and

questions related to circle geometry (Akinyemi et al., 2024; Nwafor et al., 2024; Omere & Ogedengbe, 2022). That prompted for the presentation of this article.

Circle theorems in the Nigerian Mathematics Curriculum

Following the Federal Government reforms on education and the meeting with National Council on education in Ibadan in 2005, as reported by United Nations Educational, Scientific and Cultural Organizations (2008), directed the Nigerian Educational Research and Development Council (NERDC) to reviewed the Senior Secondary School Mathematics curriculum. In respond to the development, a High –Level Policy Committee (HLPC), on curriculum development made up critical stake holders and chair by NERDC, took the guidelines for re-structuring the curriculum. In 2007, NERDC revised the old Mathematics curriculum with the intention of producing a crop of well-educated Nigerians who will be adequately equipped to function effectively in the present globalize world. The decision for the revision of the curriculum was dictated by the Federal Government of Nigerian to:

attain the number two Millennium Development Goals (MDGs), that is achieving the Universal Basic Education by 2015;

attain the targets of the National Economic Empowerment and Development Strategies (NEEDS);

produce better informed Information Communication and Technology (ICT) complaint citizens of high ethical educational standard.

The new Mathematics curriculum was fully implemented in 2012, with themes changed from six- pronged to five-pronged approach as follows:

Number and numeration, Algebraic processes, Statistics, Introductory calculus which focuses on differentiation integration of polynomials, Geometry has plane geometry, coordinated geometry, mensuration and trigonometry. Also, the Circle geometry in the new curriculum contain the following: Circle; parts of a circle, Chord properties; angles subtended by chord in a circle, perpendicular bisectors of chords and the angles in an alternate segment. There are also, Circle theorems; the angles which an arc subtends at the center is twice the angle it subtends at the circumference, angles in the same segment of a circle are equal, angles in a semi-circle, cyclic quadrilaterals and tangents to circle(s). Circle theorem is a subdivision of Euclidean geometry that includes the use of theorems, theorem converses, corollaries and axioms (Hanna, & De Villiers,2008; Segbefia,2022).

Proving Skills of Circle Theorems

Theorems are mathematical statements which can be verified using proofs. It is a mathematical judgement whose truth is established by proof. Also, it is a statement that has

been proven to be true. The reason for a statement can come from the set of given premises or from one of the four types of other premises: definitions; postulates; properties of geometry, or congruence; or previously proven theorems. Once a conjecture is proved, it is called a theorem. More so, it is a convincing argument that something is true. In Mathematics, a proof starts with things that are agreed upon, called postulates or axioms, and then uses logic to reach a conclusion. Conclusions are often reached in geometry by observing data and looking for patterns. The conclusions reached by inductive reasoning is called a conjecture. It consists of a sequence of statements, each supported by a reason, that starts with a given set of premises and leads to a valid conclusion. This type of reasoning is called deductive reasoning. Each statement in a proof follows from one or more of the previous statements. It assures that the theorem is true and remains valid also in the future and it is a logical argument that demonstrates why a theorem is true, using established axioms, definitions, and previously theorem(s) (Knill,2020).

In geometry, proof is a step-by-step use of reason argument to show that a geometrical property is true (Bruno,2023; Hanna & De Villiers,2008; Lakatos, 1976), therefore, skills in a formal proof of a theorem are as follows.

1. The geometry property is given as a statement in a word and titled" Theorem''.
2. A drawing of the geometrical shape in the theorem is made with lettering.
3. The theorem then is stated in terms of the lettering of the diagram/figure in step 2 and titled ''Given''.
4. What is required to be proved is stated in terms of the lettering of the diagram in step 2 and titled ''Required to Prove''.
5. It is sometimes necessary to draw in the diagram/figure extra lines under the titled ''Constructions''. The construction line(s) are usually drawn as dotted line(s). Here, the teacher can link step 5 to a situation in a student is required to perform the task.
6. The formal proof is titled ''Proof',' it consists of mathematical statements which uses basics geometrical properties and logical deduction reasoning. By the side of each statement, reason(s) is/are stated in bracket(s) or the basic geometric property used. The proof is completed when is required to be proved is reached. As such, it becomes a premise for geometric arguments you can use to prove other conjectures. The four common methods of geometric proofs are:

1) Paragraph proofs – a proof in which the statements and their justifications are written together in logical order in complete sentences. There are always diagrams and statements given and prove sections before the paragraph.

2) Two-column proofs - a proof in which the statements are listed in logical order in one column, and the reason each statement is true is in another column. The last statement is always what is proven.

3) Flow chart proofs - a concept map that shows the statements and reasons needed for a proof in a structure that helps indicate the logical order. Statements, written in the logical order, are placed in the boxes. The reasons for each statement is placed under that box.

4) Coordinate proofs - a proof involving midpoints, slopes, and distances on the coordinate plane.

Usiskin (1982) and Vojkuvkova (2022) advocated the use of Van Hiele Levels of reasoning/thinking levels when proving geometrical shape(s), as explained below.

Visualization or recognition (Level 0): at this initial level, students recognize and name geometric shapes based on their overall appearance, without focusing on specific properties or relationships between shapes. For example, a student at this level can identify angle(s) in a semi-circle, a square as a square simply because it looks like a square, without considering the rules or properties like equal sides or right angles. Equally, they can be saying that an egg is a circle and a diameter is a line that passes through the center of a circle.

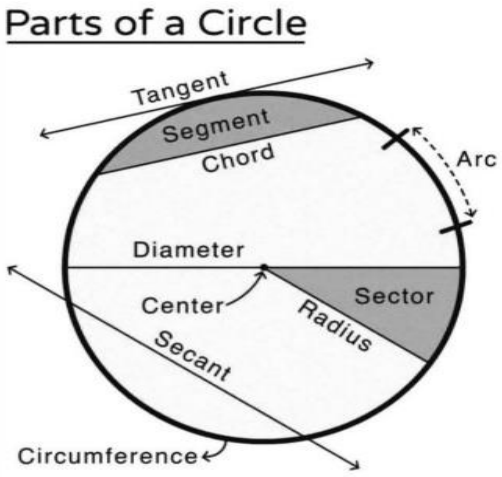
Analysis (Level 1): students at this level begin to analyse shapes by their properties and relationships among components, recognizing that shapes can be categorized based on shared characteristics empirically, but properties or rules are perceived as isolated and unrelated. They can identify that a square has four equal sides and four right angles, and they can differentiate between different types of quadrilaterals based on these properties. Meaning at this level could include: a central angle is the angle at the centre without referencing the relationship to the radii in the same circle. Also, they might identify that chord touches two different points on a circle but a diameter passes through the centre.

Abstraction or informal deduction (Level 2): this level involves understanding relationships between and among properties and recognizing that some properties imply others. Students logically order and interrelate previously discovered properties and rules giving informal arguments. Logical implications and class can see how different shapes are

related (e.g., a square is a rectangle, a rectangle is a parallelogram) and can make informal deductions based on these relationships.

Formal deduction (Level 3): students at this level can understand the role of definitions, axioms, and theorems in circle geometry. They can construct formal proofs and understand the logical structure of geometric systems. A student should be able to supply reasons for each steps in a proof or students can go beyond just identifying characteristics of shape and able to construct proofs using postulates or axioms and definition. They can use informal reasoning to solve geometric problems and prove simple geometric statements (Monique & Nfor,2023; Tong et al., 2022). A typical proving of circle theorem should be taught at this level.

Rigor (Level 4**)**: at the highest level, learners can compare different deductive systems and explore different geometries based on different axioms. They can understand the nature of mathematical systems and the foundations of geometry. They can apply advanced mathematical concepts and techniques to solve complex geometric problems and prove advanced theorems (Tong et al., 2022). This level involves the ability to apply advanced mathematical concepts and techniques.



Radius: - any interval joining a point on the circle to the centre.

Chord: - an interval joining two points on the circle.

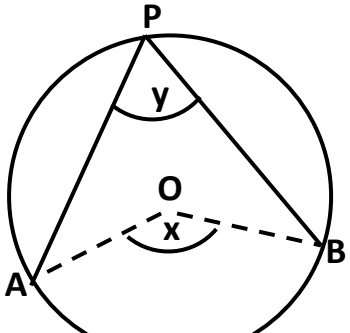
Diameter: - a chord that passes through the centre of a circle.

Secant: - a line that cuts a circle at two distinct points

Figure 1

Statement of Circle Theorems and Proofs

Theorem: the angle subtended by an arc of a circle at the center is twice that subtended at any point on the remaining part of the circumference.



Interpretation for proving of the theorem figure 2a.

Construct a circle of your choice with center O.

Locate two points A, B, on the minor arc AB.

Joint O to A and B to form $\angle AOB = x$

Locate another point P on major arc AB = y.

Joint A to P and B to P as shown in figure 2a.

Figure 2a

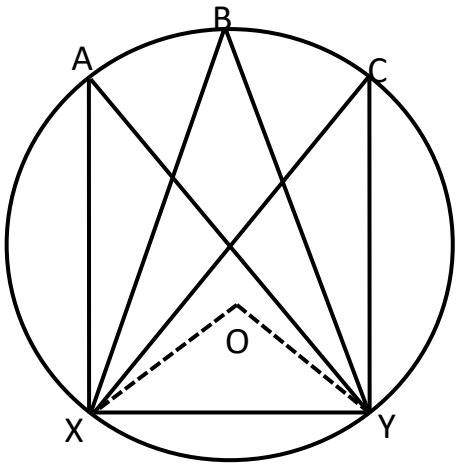


Figure 2b

Therefore, $< AOB = 2\angle APB$

$$x = 2y$$

i.e $\frac{1}{2}\angle AOB = \angle APB$

Similarly, in figure 2b, $\angle AOB = 2\angle XAY$

$$\angle AOB = 2\angle XBY$$

$$\angle AOB = 2\angle XCY$$

Steps for proving the theorem from figure 2c

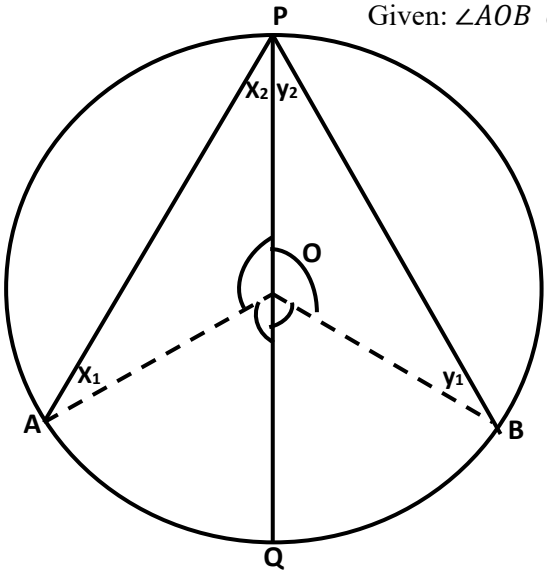


Figure 2c

Given: $\angle AOB \ of$ a circle center O, with P any point on the remaining arc.

Required to prove:$\angle AOB = 2\angle APB$.

Construction: Draw a bisector of the

Proof: $|OB| = |OB|$ (radii).

$\angle OAP = \angle OPA$ $(Base \angle s \ of \ isosceles \triangle)$.

$\angle OAP = \angle OAQ + \angle OPA$ $(External \angle s \ of \angle OPA)$

$$\angle OAP = (2x_1 + 2x_2)$$

$$\angle AOP = 2(x_1 + x_2) = 2x_1$$

$$Also, \angle BOQ = 2\angle OPB = 2y_2$$

$$\angle AOB = \angle AOQ + \angle BOQ$$

$$\angle AOB = (2x_1 + 2y_2)$$

$$= 2(x_1 + y_2) \text{ (Factorization)}$$

$$\therefore \angle AOB = 2\angle AP$$

Theorem: angle in a semi- circle is a right angle i.e 90º.

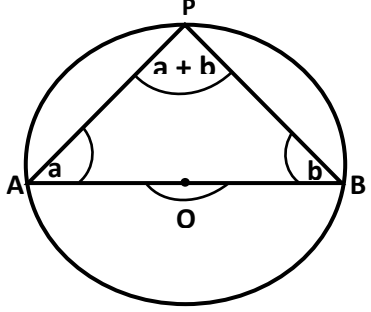


Figure 3.

From figure 3 above, $\overline{AOB}$ is a diameter which divide the circle into two equal parts and locating point P on any parts of the circumference in figure .

Given: $\overline{AB}$ is a diameter with angle center O.

$a = b(Base\ \angle s\ of\ isosceles\ \triangle\ are\ equal)$.

Required to proof: $\angle APB = 90^0$.

Proof: $\angle APB = \frac{1}{2} of \angle AOB(\angle\ at\ centre\ = 2 \times \angle\ at\ the\ circumference)$

$$\angle APB = \frac{1}{2} \times 180°$$

$$\therefore \angle APB = 90°.$$

CYCLIC QUADRILATERAL

A cyclic quadrilateral is an embedded quadrilateral in a circle and the vertices of the quadrilateral lie on the circumference of a circle. See the diagrams below.

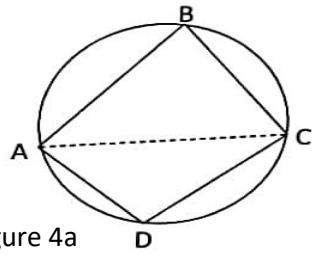


Figure 4a

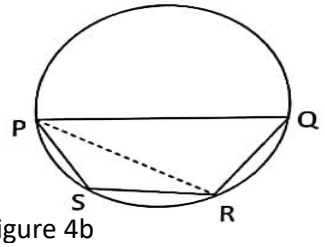


Figure 4b

ABCD and PQRS are cyclic quadrilaterals. Notice that opposite angles of a cyclic quadrilateral lie in opposite segments of the circle.

THEOREM: The opposite angles of a cyclic quadrilateral are supplementary, or angles in opposite segments are supplementary.

Note: the sum of supplementary angles is 180^0

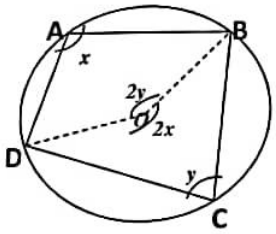


Figure 4c

GIVEN: A cyclic quadrilateral ABCD
TO PROVE: $\angle BAD + \angle BCD = 180^0$
CONSTRUCTION: Join B and D to the centre O of the circle ABCD

PROOF:
With the lettering in the above diagram,
$\angle BOD = 2y$ (angle at centre = 2× angle at the circumference)
Reflex $\angle BOD = 2x$ (same reason)
$\therefore 2x + 2y = 360^0$ (angles at a point)
$\therefore 2(x + y) = 360^0$
$x + y = 180^0$
$\therefore \angle BAD + \angle BCD = 180^0$

Adapted from Essential Advanced General Mathematics (2024)

Recommendations

1.Teachers and students should always use step by step plans instructions to prove circle theorems.

2.The proving skills for circle theorems should be targeting meaning making, relational thinking and logical abstraction. This can be done by using real life scenarios or dynamic geometric construction in figure(s) of circle theorems

Conclusion

The article is aimed at improving students' ability to accurately state and apply the key components of a circle to prove circle theorems such as the inscribed angle theorem: the angle at the centre being twice the angle at the circumference, angle in a semi-circle and cyclic quadrilateral. The paper identified the connecting theorems to diagrams relevancies based on the given geometric information to be used, showcasing proving skills and logical arguments using deductive reasoning to prove circle theorems. This involves starting with given information, applying relevant definitions and theorems, and arriving at a conclusion through a series of logical step-by-step proofs in a clear and organized manner, with each step justified by a valid reason. It aimed at promoting the student's overall proficiency level in proving circle theorems in internal and external examinations. The skills demonstrated the basics for advanced understanding and hoping could further develop their skills, such as strengthening their deductive reasoning or improving their ability to apply theorems to more complex problems and daily life.

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